TECTONIC FORM MAKING

Form as Diagram of Forces: The Equiangular Spiral in the Work of Pier Luigi Nervi

Thomas W. Leslie, AIA
Iowa State University

"The form, then, of any portion of matter, whether it be living or dead, and the changes of form that are apparent in its growth, may in all cases alike be described as due to the action of force. In short, the form of an object is a 'diagram of forces,' in this sense at least, that from it we can judge of or deduce the forces that are acting or have acted upon it: in this strict and particular sense, it is a diagram, —in the case of a solid, of the forces that have been impressed upon it when its conformation was produced, together with those that enable it to retain its conformation."

—D'Arcy Wentworth Thompson, *On Growth and Form*, Introductory. (1)

D'Arcy Thompson's On Growth and Form contains within its literate prose the profound suggestion of a morphology based on empiricism and mathematics; a powerful challenge to our established architectural traditions of style and composition. Thompson, of course, had no such motivations in writing, and despite his occasional use of architectural (or more commonly structural) metaphors it seems unlikely that his work's translation into the field of architecture would have held more than a passing interest for him. Yet his masterwork has an unavoidable relationship to the work of the designer, his examinations of the formation and performance of matter in biology containing lessons and examples that contain inevitable consequences for architecture, in which form is also at least in part determined by both assembly and function. In architecture the conscious intelligence of the designer takes the place of the latent intelligence accumulated by natural selection; perhaps more importantly, the operations of organic and constructive processes are likewise related but distinctly conceived. The organism grows by gradual accumulation over its life, the architectural structure by relatively instantaneous assembly at its very inception. The relationships between organic morphology and architectural form, therefore, can be expected to show dialogical affinities, rather than determinant linkages. In the example of Italian *constructeur* Pier Luigi Nervi (1892-1979), these affinities emerged with striking clarity in a set of long-span works, demonstrating the potential richness of an empirical design process and hinting at a limited though important correspondence between the world of the organic and that of the constructed.

Nervi's roof forms suggest that the strongest link between the organic and the constructed exists not at the level of visual or formal representation, but rather in the deeper structures of geometry and mathematical patterning. While metaphor and the imitation of striking natural forms may be architecturally tempting, Nervi's work suggests an 'organic' that lies in the relationships between constructive and structural processes, and the deployment of material to best enable these. This theme, which permeates Thompson's work, connects our constructions to the natural world through the notion of design as arrangement of resources for benefit, not design as the construction of spatial or graphic experience. The resulting forms, whether they are natural or man-made, display patterns and formations that we recognize as beautiful or engaging not simply because of their visual proportions but rather because of our innate recognition of their mathematical efficiency, their logic, and their patterning. A comparison between these two realms reveals the mechanisms of this 'organic' beauty in simple geometry, and suggests both the potential for and the limits of the conceptual link between organic and structural morphogenesis.

MECHANISM AND TELEOLOGY—THOMPSON'S VIEW ON GROWTH AND PERFORMANCE

The place of On Growth and Form in the history of biology demands clarification, primarily for its critique of Darwin's reliance on function to explain evolutionary formation. The traditional teleological "argument from design," used as an attempt to scientifically prove the existence of an intelligent Creator, was arguably in Darwin's case only slightly modified, to substitute the iterative logic of evolution for a divine plan. The overall thrust of Thompson's argument, all too often lost in architectural references, was an attempt to explain organic formation through the question of motive or efficient causation. Whereas Darwin saw evolution as primarily concerned with outcomes, Thompson was primarily concerned with means—no matter how advantageous a hypothetical adaptation might be, its formation must in Thompson's view be physically achievable given the organism's milieu. The resultant "weaving together" of the "warp and woof...of mechanism and teleology" took its inspiration in part from Aristotle's The Parts of Animals. As Thompson points out, Aristotle's relationship between final and motive causation may be best described with an architectural metaphor: "...the house is there that men may live in it; but it is also there because the builders have laid one stone upon another." (2) Form may thus be seen as the resultant of these two sets of forces. In the case of Nervi, the description of these forces being interwoven will be particularly apt, as his process suggests a fluid integration of function, construction, and structure.

While Thompson delved into problems of teleological causation at the level of performance—notably in the chapter "On Form and Mechanical Efficiency," a direct reference to Aristotelian causation—the dominant theme in On Growth and Form is therefore that of process, the formation or assembly of organic structures. Throughout numerous examples, Thompson demonstrated that process invariably plays a role in the determination of organic form, and that efficiency in formation (motive cause) is as determinant a role in morphology as efficiency in performance (final cause). Underlying nearly all formative processes, Thompson finds mathematical principles or algorithms that organize and streamline growth processes, and it is here that he found startling similarities in organic forms across and throughout species.

The equiangular, or logarithmic spiral, was Thompson's most striking example, both for its compelling geometry and its widespread occurrence in diverse phenomena. This spiral is the product of a simple geometric algorithm, in that it consists of the path traced by a moving point that is simultaneously sweeping around and accelerating away from a fixed center. (3) The resulting shape is at once self-similar—any segment of its curve is exactly proportional to any other—and constant in its local angular relationship to its central point. The spiral has long been of interest mathematically due to its bizarre properties, among which is its tendency to self-reproduce through transformative actions; for instance, the path traced by a beam of light sourced at the center of such a spiral will itself be reflected into a logarithmic spiral. (4) Crucial to our study, and noted at length by Thompson, are the spirals "gnomonic" properties, in which any portion of the spiral, when removed or added, produces a resultant shape proportionally similar to the original. The spiral's most striking architectural property, its relationship to the Golden Section, results directly from its gnomonic tendencies. Because of the spiral's self-similarity, a series of nesting rectangles (or, for that matter, any other regular shape) laid out with vertices at regular polar intervals along the path of an equiangular spiral will necessarily be similar to one another and to the overall composition. In this process the addition of the gnomon shape will create a new figure proportionally similar to the old. In the case where each additive shape is a square, the resulting figure will be a rectangle with proportion 1:1.618034, the so-called Golden Ratio. (5) The spiral's self-similarity or 'ratio repetition' also tends to produce proportions based on the Fibonacci series—the series of numbers in which each term is the sum of the previous two:

1 1 2 3 5 8 13 21
34 55 89 144 ...

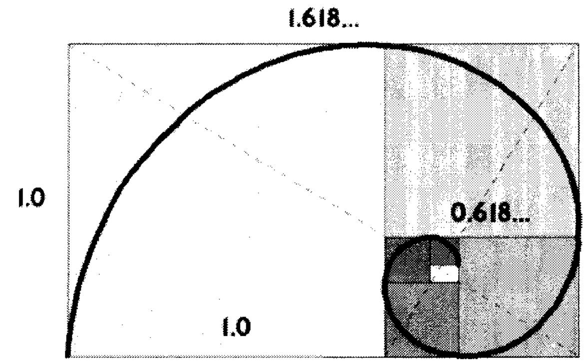

Fig. 1. The Equiangular Spiral. This particular instance manifests the proportions of the architectural Golden Section.

For Thompson, however, the spiral in all its elegance is most simply a signature of staged, exponential growth, or, rather, accumulation. As he points out, the equiangular spiral primarily exists in dead matter not living, as it is not a result of constant growth but is rather a by-product, typically the result of an organism accreting or excreting matter over time. This temporal aspect is a key element in the spiral's manifestation, as the activity of adding to a shell, or horn, is necessarily based on the previous assembly's form and scale. Such phenomena as shells, horns, or teeth all share this process of development, and Thompson therefore deduces that these spirals come about as the results of highly efficient algorithms—the incremental production of dead matter in constant relation and proportion to forms already produced. Put in Thompson's language, the result is growth in size without change of shape. In cases such as the nautilus shell, this can be simplified even further, in that the shell's fabricational instructions can be stated simply as the result of constant accretion of the outer shell at a given, positive angle to the existing, inner shell. It is, in fact, unlikely that such a process could create any form other than an equiangular spiraL. (6) The double curvature of the resultant shell or horn is not only a result of the constructive algorithm; it is an inherently efficient use of the material for structural purpose. Furthermore, the proportionally repetitive nature of the shell deploys material according to its distance from the center—the shell is always thickest at its exterior, while the horn adds additional material at the base. In each case, the spiral provides a structurally efficient shape that increases its performance as it grows.

This algorithmic rigor, evident throughout Thompson's examples, presents a powerful challenge to architectural form making. Through Aristotle's analogy to house building one can see obvious parallels—architecture is concerned with both motive and final causes, however forces of commodity often overtake those of assembly and function. Our resources in terms of materials and constructional algorithms are not always determined by necessity, and thus the rigor that accompanies natural morphology is not always present in architectural situations. Nevertheless, in situations of extremity—long spans, tall structures, etc.—processes of making and shapes derived from static models become more determinant, and the resulting forms more closely approach the ideal geometries dictated by physics. In particular, the following investigation of process in the long span work of Pier Luigi Nervi will demonstrate the confluence of final and motive causes under circumstances of extreme constructional and structural duress, into structures that have precisely delineated parallels with the organisms noted above. This examination will elucidate an architectural approach that resists the prejudice of the visual by rigorous adherence to the efficiencies of constructional algorithms and structural principles. That the visual results of this disciplined approach are nonetheless visually engaging suggests that mere composition does not fully do justice to our sense of beauty, which can be provoked and engaged by intuitively recognizable mathematical patterns as well as emotive, expressive form.

MEANS AND MOTIVE—THE DEVELOPMENT OF THE LAMELLA SURFACE IN NERVI'S ROOF FORMS

Nervi's work represents precisely this "Thompsonian" viewpoint, in that his work demonstrates an expression not only of the functional imperatives behind each project but also the constructional processes involved—the 'motive' in addition to the 'final' causes. Nervi recognized the potential for reinforced concrete to meet the needs of new programmatic requirements and to create a new sense of architectural design that transcended mere aesthetics. Writing in 1963, he defined his work to that point as an overall manifestation of "structural architecture," whose major points he summarized as follows:

"I believe the essential conditions of structural architecture to be as follows:

1. It must give a convincing answer to a real and authentic static necessity and be determined by it.

2. A static constructive scheme should become visible and comprehensible inside and outside.

3. It must express frankly the material with which the structure is executed and find in the technological characteristics of the material itself the sources and ways, as well as the details of its architecture." (7)

Here we find a succinct expression, in architectural terms, of Thompson's "warp and woof" integrated into a singular philosophy of design. If Nervi's first and last points can be seen as analogous to final and motive causes, respectively, it is important to note the subjective nature of his second. Nervi's trifold training as engineer, architect, and constructor is very much in evidence, as it is apparently not enough to merely arrive at the most efficient solution, although that is clearly part of the designer's charge. One must also strive for a communicative aspect, based in the clear revelation of the forces at work. Thompson's notion that "form" may be analyzed as a "diagram of forces" is in Nervi's work thus reversed—"form" may also be synthesized from precisely such a static diagram as well, through the means and expressed in the visual language of construction. This process may be seen most clearly in Nervi's development of the 'lamella' structural principle in a series of works executed from 1935 to 1960.

In 1935, Nervi received a commission for a series of aircraft hangars, to be built at Orvieto for the Italian Air Force. It is notable that this commission was won not by reputation, but via competitive bid, and therefore the scheme proposed by Nervi is one of extreme simplicity and efficiency. To span the long distance required by the size of the aircraft, Nervi proposed a lamella shell structure; essentially a series of directionally opposed intersecting arches set diagonally to

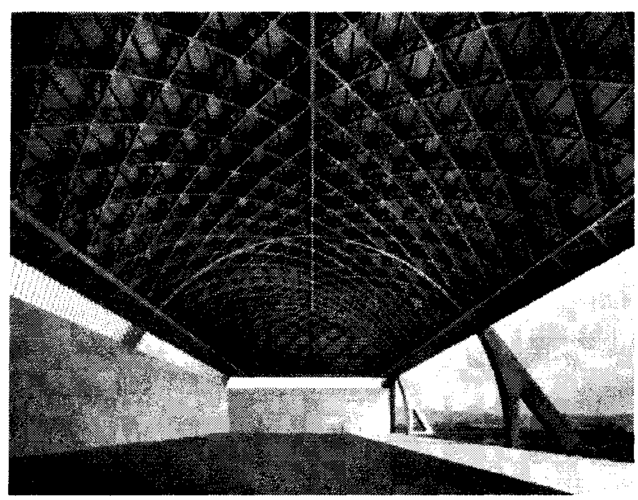

Fig. 2. Aircraft Hangar at Orvieto. Pier Luigi Nervi, 1942.

the rectilinear volume below. The resultant diamond pattern can be thought of as either a space-frame wrapped around a cylindrical volume, or as a barrel arch with much of its dead weight removed, in either case combining the actions of latticed roof structures with an arched shape and allowing an extraordinary span/weight ratio. Because of lamella pattern's inherent triangulation, each joist is capable of carrying both gravity and lateral loads, while their proximity and connections to one another allow loads to be distributed throughout the network, rather than carried by a single member. Deployed along curved surfaces, lamella joists offered the load-carrying abilities of a long-span arch with the lightweight of thin-shell concrete, or thought of another way, the possibility of long-span compression members with very short unbraced lengths. (8) Reflecting the state of construction and design technology at the time, the hangars were built of intersecting circular cross sections. Parabolic sections would have been structurally ideal; however the consistent shape of the radiused roofs approached the proportions of the perfect parabola while allowing each bay to be of uniform dimensions—a notable compromise between forces of assembly and performance at the largest scale.

The first pair of these hangars to be built were composed of poured-in-place concrete, using removable formwork to obtain narrow arch profiles with hollow-block roofing. While the material savings from the structural solution were significant, Nervi recognized inherent flaws in both the process and final structures. As the next phase was also put to competitive tender five years later, Nervi refined both the structural scheme and the constructional process. The ribs of the next set of hangars were prefabricated, saving immense costs in labor and formwork. Forming these ribs on the ground allowed more intricate casting, which meant that ribs not subject to large shear or bending forces could be fabricated as latticework, lightening the structures' dead loads considerably. Combined with a series of design changes that allowed a simpler pier scheme, the second set of hangars proved dramatic savings in material and construction time before being destroyed by retreating German forces in WWII. The development of two distinct fabricational methods for the roof structures reflects the evolutionary nature of Nervi's process. The second iteration of hangars maintained the structurally efficient geometry of the original designs while reflecting a more efficient construction method—a precise parallel to Thompson's version of natural selection, in which algorithms are 'chosen' for their clarity and material economy. (9)

Nervi did not often explore the static and constructional efficiencies of the lamella shell system in subsequent work, as the majority of his long span work was to focus instead on the potential for corrugated or folded plate elements. However, two ideas from these early hangars run through virtually all of his later work, namely the use of precast beam elements and "coffered" roof spans where structural depth is maximized along lines of isostatic force. A third consistent element in Nervi's work, the use of "ferro-cement" formwork left permanently in place, addressed an intense shortage of timber in postwar Italy, and the desire to maintain control over the internal appearance of the expressed structure. The problem of formwork was fundamental to the nature of concrete construction for Nervi, as previous designers had been limited by the general reliance on timber or steel formwork to contain concrete while it cured. This led, in Nervi's mind, to an inarticulate expression, as it necessarily adopted the forms and markings of timber construction, not those intrinsic to the concrete itself. Ferro-cement formwork was made by injecting rich concrete mortar into molded wire mesh—while labor intensive, this process led to a very thin, durable panel that took advantage of the steel mesh's capacity in bending. More importantly, the ferro-cement panels could be made to adopt any shape desired, particularly curvilinear geometries reflective of concrete's poured nature and the gently arcing lines of static force in slabs and arches. (10) These shapes could be hand-molded with accuracy by forming the wire mesh over pre-set jigs, allowing multiple units to be made reliably and quickly. The use of ferro-cement eliminated the need for timber and in fact took advantage of the region's one major economic resource, an abundance of inexpensive labor.

In 1948, Nervi competed for and won the contract for a new Exhibition Hall in Turin. With fewer than eight months for construction, and a program requiring spans of over 100 meters, Nervi was again faced with the need to produce a scheme of extraordinary constructional efficiency and he therefore proposed a series of poured in place concrete buttresses supporting a corrugated shell of precast elements. The roof elements, made of ferro-cemento, repeated a structurally efficient folded plate module hundreds of times over the surface of the roof. A series of intermediate diaphragms gave each unit its own integral stability, while providing adequate contact area to ensure overall monolithic performance. The result was a finely grained roof structure, elegantly collected at its base by a series of fan vaults that added to the hall a sense not only of scale, but also of process—a visualization of forces both constructive and static. (11)

Nervi produced a number of long span halls based on the Turin model; however, a relatively small apsoidal hall at the end of the main exhibition space is of particular interest to the study of Thompson's morphogenetic theories. This apse had been conceived as a response to the oddly shaped site, a semi-circular space that extended the main exhibition area at a smaller scale. Because of the static geometry of the main hall, there was no way to carry the logic of the corrugations into this element, and instead Nervi devised an ingenious system of radially arrayed coffers to transmit the weight of the dome to a colonnade beneath. These coffers were formed of diamond-shaped ferro-cemento pans, in fourteen standardized shapes to leave a geodetic pattern of ribs on the underside surface. The ribs were thus arrayed in an efficient structural pattern, following lines of gravitational and torsional stress down to the colonnade. While the final effect is visually striking, the logic of the construction and structural performance of this system is perhaps more compelling, While forming an efficient, monolithic structural system of in situ concrete, the ferro-cemento pans ensured a consistent appearance based on the tight quality control achievable by precasting. The ability to deploy a set of mass-produced elements as both formwork and finish material allowed dramatic cost and schedule savings in the construction of these works. It is apparent from the final appearance of the dome, however, that Nervi had not fully grasped the spherical geometry of the structure, as the proportions of the pans vary widely toward the top of the dome. This is not merely an aesthetic shortcoming; it meant each family of pans required a separate set of jigs for their formation. In addition, the irregularity of the ceiling ribs violated the angular order of the lamella principle, meaning that the natural redundancy of the system was not fully exploited.

In subsequent designs for the Ostia Beach Casino, the Terme di Chianciano, and most notably the Palazetto dello Sport for the Rome Olympics, Nervi improved on this method with dramatic results. If the Hangars of the 1940s were pure lamella roofs, essentially wrapping a diamond pattern around a cylinder, these later projects represented a rotational interpretation of the lamella principle. Instead of a series of diagonal arches on a rectilinear plan, the roof structures of these projects can be seen as diagonal arches based on a polar plan, in other words, with regular angular relationships and proportions based on their radial distance from the center. The resultant shapes provided an efficient distribution of gravity loads along the surface of the roof, while at the same time providing a geometrically based resistance to lateral and torsional forces due to their inherent triangulation—avoiding the inefficiency of the Turin apse.

The best known of Nervi's rotated lamella domes is the small Olympic arena built for the 1960 games in Rome. While the larger indoor space, the Palazzo dello Sport, consisted of a prefabricated trough system, analogous to the first Turin Exhibition Hall structure composed on a polar grid, the Palazetto dello Sport represented the purest realization of the radial pan system. From the outside, the overall structural action of the dome is clearly expressed. The dome's pan joists are gathered at the structure's perimeter into a series of fan vaults, which are in turn supported by 36 Y-shaped buttresses that take gravity and thrust loads to ground foundations. The exterior expression itself, combined with the careful placement and junc-

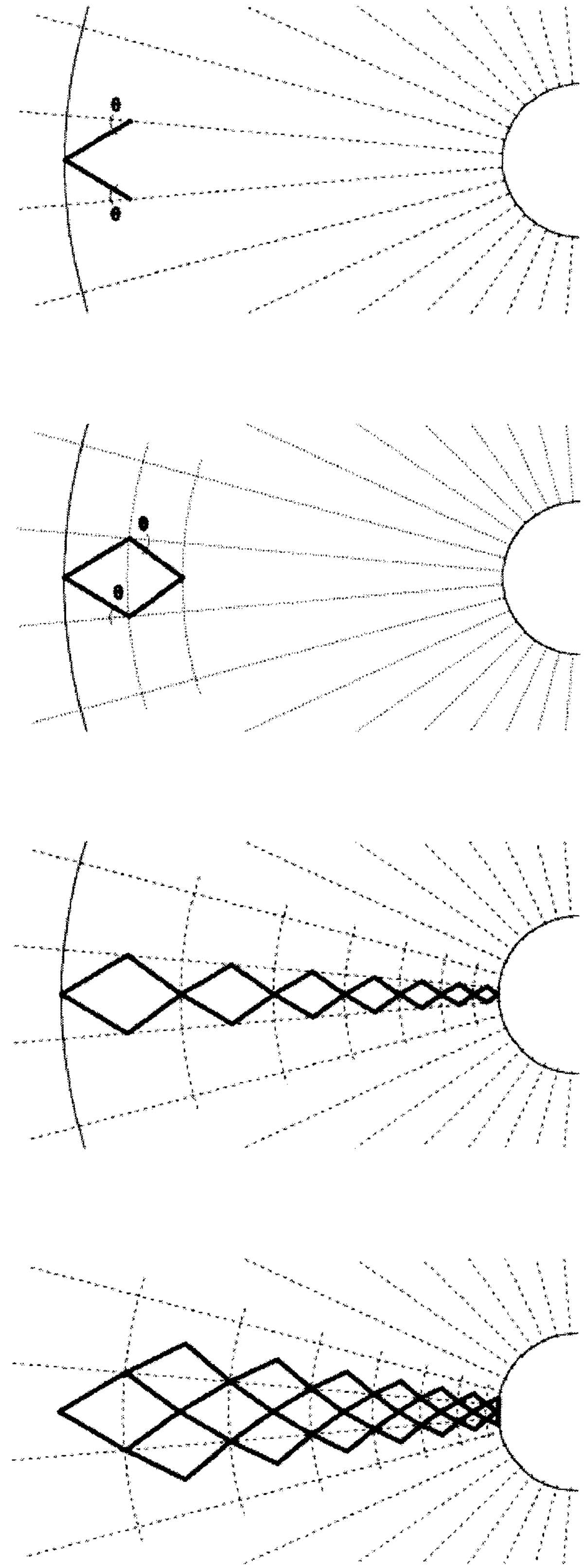

Fig. 3. Proportioning system used by Nervi on the lamella dome projects.

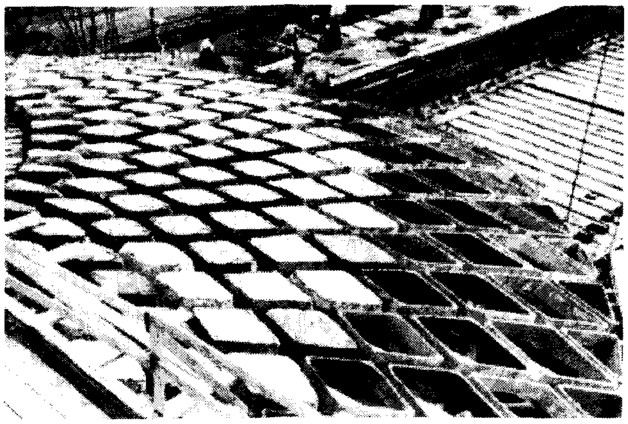

Fig. 4. Constructional system used on the lamella dome projects. Pans of ferro-cemento *form a permanent formwork for the poured concrete dome.*

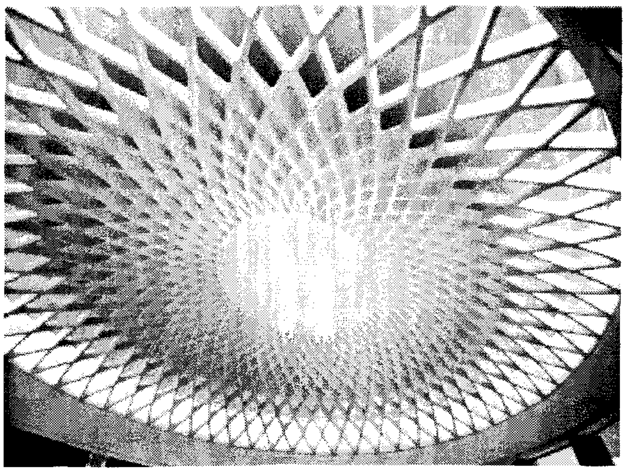

Fig. 5. Terme di Chianciano, Pier Luigi Nervi, 1952. Roof structure showing pattern of ferro-cemento formwork.

tion of the glass curtain wall enclosure, provides a clearly conceived and lucidly expressed resolution of the forces involved, using job-cast members whose forms are manifestations of the dome's static vectors. Between the fan vaults, where enclosure is required but where the dome's loads are not being carried, the concrete shell is pulled upward into an eyebrow, admitting daylight to the concourse area while indicating that the concrete here is carrying no load from the dome above. From without, the interplay of the sharply angular buttresses with the undulating circumference of the dome is an essay in static and constructional fluency, showing an ordered structural and enclosure system whose patterns break down the mass of the building into human scale.

The elegance of the Palazetto's exterior is, however, far surpassed by its striking interior, whose genesis merits careful examination. In 'bending' the lamella principle into a radial form, Nervi faced the

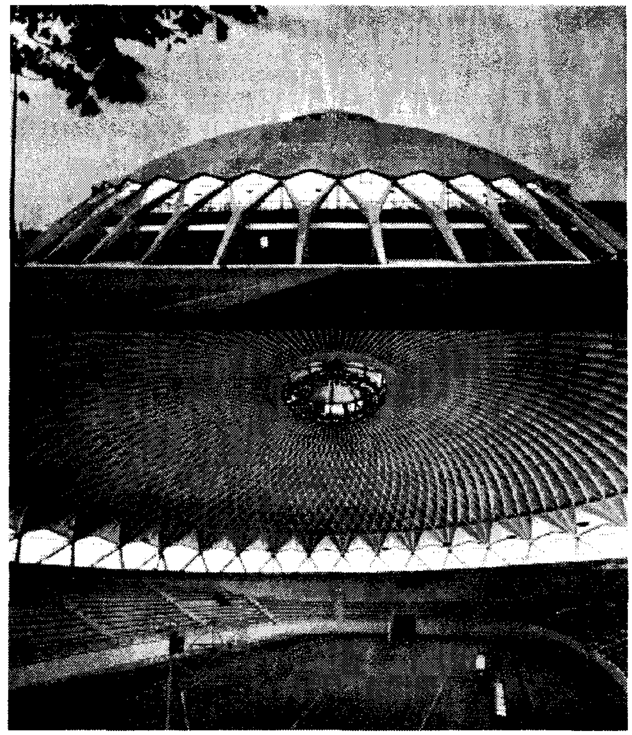

Fig. 6. Palazetto dello Sport, Pier Luigi Nervi, 1958. Exterior showing y-buttresses, and interior showing pattern of lamella dome.

geometrical problem evident in his earlier work, in that the diamond modules of the roof necessarily changed in scale based on their distance from the center. In his earlier experiment with this principle the results had been inconsistent, necessitating modules varying in not only size but also in proportion to fill the domical surface. At the Palazetto, this problem was solved via a simple geometric algorithm, with intriguing results.

Exactly how Nervi's proportioning system developed is not known, however we can speculate on how it may have been done. This elucidation of Nervi's process show an affinity to Thompson's reliance on motive causation to explain form, as it stems from an economic requirement to minimize variation in geometry throughout the system. Such consistency allowed a repetition not only of the molds used to bend the wire cages on which the ferro-cement was placed, but also of the jigs used to make the molds, as the angles subtended by each edge could be kept consistent throughout the project. In addition, reinforcement placed in the resulting troughs formed by the edges of the pans could be designed to meet at the same angle throughout the project, allowing mass production of steel connections between embedded reinforcing bars.

The essential problem was thus to ensure a full tiling of the domed surface with similarly shaped and consistently proportioned precast elements, ensuring that each one would subtend consistent angles throughout the dome. In order to do this, one can start with an external perimeter, and an assumption about the inscribed angle of each pan—in the case of the Palazetto, 3-1/3 degrees.

Next, the angle of incidence for each joist is figured, based on the maximum size achievable for each pan, and on allowable stresses in each joist. In the two limiting cases, the angle of incidence could be 90 degrees, in which case the roof would be configured as a simple radial arch scheme, or 180 degrees, which would suggest a monolithic concrete shell. The resulting intersection of joist and circumference determines the setting out point of the next half-diamond pair of joists, essentially reflected through the newly derived circle. This process is then continued until a dimension arbitrarily near the center of the dome is reached; based on a balance between the weight of the dome's cap and the decreasing size of the pans beyond the point of fabricational convenience. A second series of pans based on the intermediate dimensions of the first series completes the fabric of a typical radial bay. Each joist is formed by the downstand edges of adjacent pans, which form a void into which reinforcement and concrete will be placed. When this typical bay is then rotated about the center to form a complete dome, the self-similarity of each joist to one another, and the consistency of the joists' angular relationship with the circumferential series give rise, predictably, to a series of interwoven logarithmic spirals, traced both by the joists themselves, and by the centers of each joist pan.

That the spirals should be manifest in such an endeavor should not be at all surprising, for the processes of assembly involved here are similar to those undertaken by the Nautilus, or indeed any of the organisms cited by Thompson as constructors of equiangular shells. In each case, an element composed of dead material—whether secreted or formed—is produced adjacent to a geometrically similar, gnomonic element with only a regulated change in scale to differentiate one from another. The role of time and sequence is important to the assembly of each—although in the Nautilus the process occurs from the inside out, while at the Palazetto the construction proceeded from the perimeter inwards. Through self-similarity, gnomonic geometry, and carefully ordered process, the pans in the Palazetto and the shell elements of Nautilus both manifest the physical documentation of an ultra-efficient constructional algorithm—essentially stated as the performance of a repetitive fabricational process in series, and with a constantly changing scale coefficient.

Intriguingly, the roof of the Palazetto bears a resemblance to another instance of the spirals in the natural world, namely the complex interweaving of florets in leaf formation. This process, known as spiral phyllotaxis, is covered in a chapter of Thompson's book that is usually abridged, both for its lack of clarity and Thompson's apparent frustration that science had not yet fully explained the process. The mathematics that govern the growth and patterning of leaves and florets produce curves and patterns similar to those of the Nautilus, albeit overlapped and internally interactive in complex ways. Like the Palazetto, like the Nautilus, phyllotaxis is an immediately visible example of the role played by geometry in both assembly and structure, in that the spatial organization of its constituent elements is a result of efficient algorithmic programming, here in multiple dimensions.

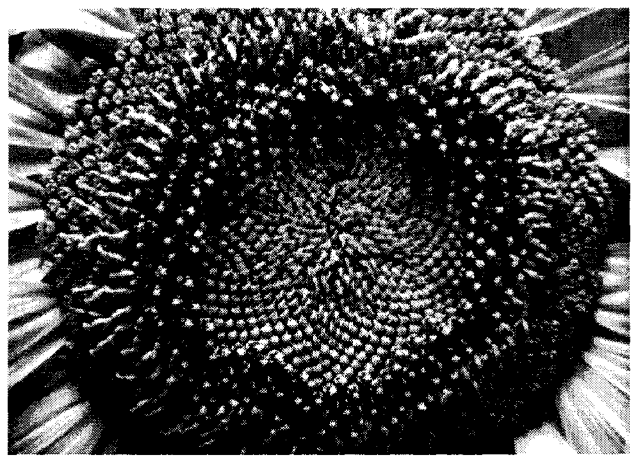

Fig. 7. Sunflower capitulum.

Most immediately analogous to Nervi's system are the patterns of seed formation in the capitula of sunflowers or daisies. (12) As Thompson explains, these florets—or 'primordia' in phyllotaxical terminology—are produced at regular intervals at the capitular edges, gradually migrating toward the center while constantly increasing in size. That they maintain their rough shapes while so doing influences the geometry of the overall scheme, producing vibrant spiral patterns; however there appears to also be an algorithmic process to their movement and development that regulates the arrangement of the primordia into these carefully ordered patterns. Known as 'parastichial spirals', these patterns of florets maintain a recognizable allegiance to the previously noted Fibonacci series. The series is a well-known signature of population growth, most famously as the number of offspring counted during regular intervals of the reproductive cycles in rabbit populations. In the case of phyllotaxis, the mathematics of the series are found in the number of parastichial spirals in each capitulum. In general, the number of spirals counted clockwise and counterclockwise will be successive numbers in the Fibonacci series. There is tremendous consistency within species regarding these numbers—in daisies, for example, the numbers [21, 34] are invariably found, while in sunflowers the numbers are [34, 55]. (13) This asymmetric pattern is related to the geometry of each primordium, which unlike the pans in the Palazetto subtend different angles at each of their four vertices. A clue to the appearance of the Fibonacci series may lie in the universal appearance of a third logarithmic spiral—more tightly wound and not visually apparent, formed by a continuous curve connecting each primordia in order of appearance. These lines are suggestive because of the geometrically self-replicating nature of the equiangular spiral and the connections between its mathematics and the Fibonacci series in geometry. It is possible, therefore, that the generative spiral is the shape based on capitular growth algorithms, while the parastichial spirals are simply resultant manifestations of the shape's tendency to self-replicate.

We have, then, three instantiations of mathematical logic in the physical world, each of which can be seen as the formal manifestation of simple—though complexly interactive—procedures of growth and construction. The Nautilus' reliance on a one-dimensional 'program' for sequential growth produces a single equiangular spiral to which each cell relates directly. The sunflower and the Palazetto each rely on multi-dimensional 'programs', and therefore the spirals manifest in each are the result of interactions between individual cells or primordia. In all three cases, the algorithms involved produce shapes that are similarly proportioned though differently scaled, in sequence and in relationships that are regular and gnomonic. In all three cases, the resultant forms are precise 'diagrams' of the forces of assembly involved, records of the constructive logics inherent in the assemblies' codings, whether organic or architectural. The human eye recognizes these equiangular relationships, which engage our sense of beauty through their regular, necessarily harmonic proportions and complex patterning.

It is here that one turns back to the Palazetto in order to find the 'parastichial order' of Nervi's system and thus, perhaps, to confirm a formal link between the lamella system and the organic world. And, of course, we are profoundly disappointed when we realize that the roof's order is the very ordinary [108, 108]—not only a pair of non-Fibonacci numbers, but also not even a sequential pair. In fact the examination of both patterns reveals that while the sunflower maintains a dynamic inequality in its rotational pattern, based on two different convergence angles in each primordia, the Palazetto demonstrates both local and rotational symmetry—with each primordia demonstrating the same convergence angle in either direction.

And yet, this is precisely what we should have expected, for Nervi's roofs share only limited algorithmical similarities with the processes of phyllotaxis. The roof of the Palazetto was constructed of ferro-cemento pans assembled in their final shapes, while primordia emerge and grow within the fabric of the capitulum. Nervi's process relied on molds and jigs, and the economies gained from re-using these numerous times. The roof pans therefore are consistently sized within each circumferential 'family', while the primordia are all sized differently depending on their time of emergence. Essentially, the algorithms of Nervi ensure maximum similarity in size and shape over the surface of the dome, whereas those of the sunflower ensure maximum density for dynamically changing elements. Rotational asymmetry appears in the sunflower as a signature of constant, incremental additions, while the symmetry of the Palazetto indicates its regularity and simultaneous configuration. With no change over time, and no dynamic elements, we should therefore expect that the Fibonacci series, characteristic signature of growth, should in fact be absent from the concrete roof.

There is, too, a fundamental difference between the two sets of patterns, in that the generative logarithmic spiral in phyllotaxis (the tightly wound connection between each primordia) is absent in the roofs of Nervi—or rather, it and its generative tendency are replaced by a series of concentric circles. These reflect the substitution of constant growth in sunflowers by the system of simultaneous fabrication and parallel assembly in the Palazetto. The Palazetto roof can thus be considered as a capitulum formed entirely by constructive processes, assembled rather than grown, static rather than dynamic. There is no root justification for asymmetry—in fact there is an economic requirement for constant sizes and angles throughout the construction. The crucial parallels between the two systems therefore lie not in their precise geometric correspondence, but rather in their reliance on fundamentally simple algorithms and the resulting manifestations of geometrical figures that are signatures of staged, modular assembly. The subtle yet critical difference of asymmetry and the parallel appearance of the Fibonacci series in the organic model are evidence that the two systems are formed by vastly different processes. The relationships between the patterns in these roofs and those of spiral phyllotaxis are, therefore, neither perfect nor merely coincidental. They are instead examples of similar underlying mathematical principles instantiated under different conditions, with results that suggest continuity between the organic and the architectural while discretely clarifying where these processes logically differ.

That Thompson should have so underappreciated the subject of phyllotaxis is thus doubly unfortunate, for it is precisely at the level of growth and assembly that the distinction between these examples is most evidently expressed. Nervi's roofs demonstrate a constructional efficiency based in the mechanical mass-production of static elements, while sunflowers show us an equally striking example of organic efficiency based in continual growth. That there is great beauty to be found in each example is hardly surprising, given the elegance of the mathematics involved. However, what seems notable here is that each system contains a legible expression of its own constitution, a systemic and thorough marking of its own generative logic. It is striking that the intuitive sense in each case can be so profoundly touched—Nervi's roofs "appear" structurally correct, the sunflower "appears" to be a natural outgrowth of its organic processes. In each case, the form of the material is a direct, easily apprehended manifestation of the processes at work. They are thus the "warp and woof of mechanism and teleology" made corporeal as signatures of the modes of making, diagrams indeed of forces at once beyond our comprehension yet within our understanding.

Perhaps the most intriguing suggestion of this parallel is that the most profound similarities between the architectural and the organic take place precisely in the realm of causation—that is, in the orchestration of motive and final cause, the 'how' and the 'why' of the phenomena involved. Nervi described these as the "essential problems" of architecture—the structural (final), the constructional (motive) and the architectural (expressive). That Nervi included this third element, subjective and slippery though it might be, was indicative of a humanist spirit that pervades his works of technical rigor. The visual joy offered by these roofs, Nervi wrote, was evidence of "the mysterious connection between the laws of physics and our esthetic sensibility." (14) In much the same way, Thompson noted that "whatsoever is beautiful and regular is also found to be most useful and elegant." (15) Here we find the ideal of organic beauty based not in composition, but rather in the humble revelations of the processes that conduct and govern the worlds of the natural and the constructed.

"Here we are in the kingdom of numbers, and each number is a flower. It joins another one at the right place to bloom into a bouquet. The numbers and the forms authorizing the slow elaboration and the skillful development of flowers and crystals always generate the same designs. These designs represent another face of light which is total architecture; they are the consequence of a subtle, prodigious planetary geometry wherein randomness means aberration."

—Roger V. Jean, *Phyllotaxis: A Systemic Study in Plant Morphogenesis.*(16)

NOTES

[1]D'Arcy Wentworth Thompson. *On Growth and Form.* (Cambridge: University Press, 1917). 11.

[2]*Ibid.* 6.

[3]Vagn Lundsgaard Hansen. *Geometry in Nature.* (Wellesley, MA: A. K. Peters, 1993). 5.

[4]William Allen Whitworth, B.A. "The Equiangular Spiral, its Chief Properties Proved Geometrically." *The Oxford, Cambridge, and Dublin Messenger of Mathematics,* vol. 1. (Cambridge: Macmillan and Co., 1862). 5.

[5]Robert Dixon, "The Mathematics and Computer Graphics of Spirals in Plants." *Leonardo* Vol. 16, no. 2. 1983. 86-90. See also the provocatively titled David Ward, The *Penguin Dictionary of Curious and Interesting Geometry* (London: Penguin Books, 1991) 67-68 for an overview of the spiral's other properties.

[6]Peter S. Stevens, *Patterns in Nature* (USA: Atlantic-Little, Brown, 1974, 81-93 provides a clear overview on the inevitability of the spiral in this type of growth.

[7]Pier Luigi Nervi. "Some Considerations About Structural Architecture." *Student Publications of the School of Design.* (Raleigh, NC: North Carolina State College, 1963). Vol. 11, no. 2. 43.

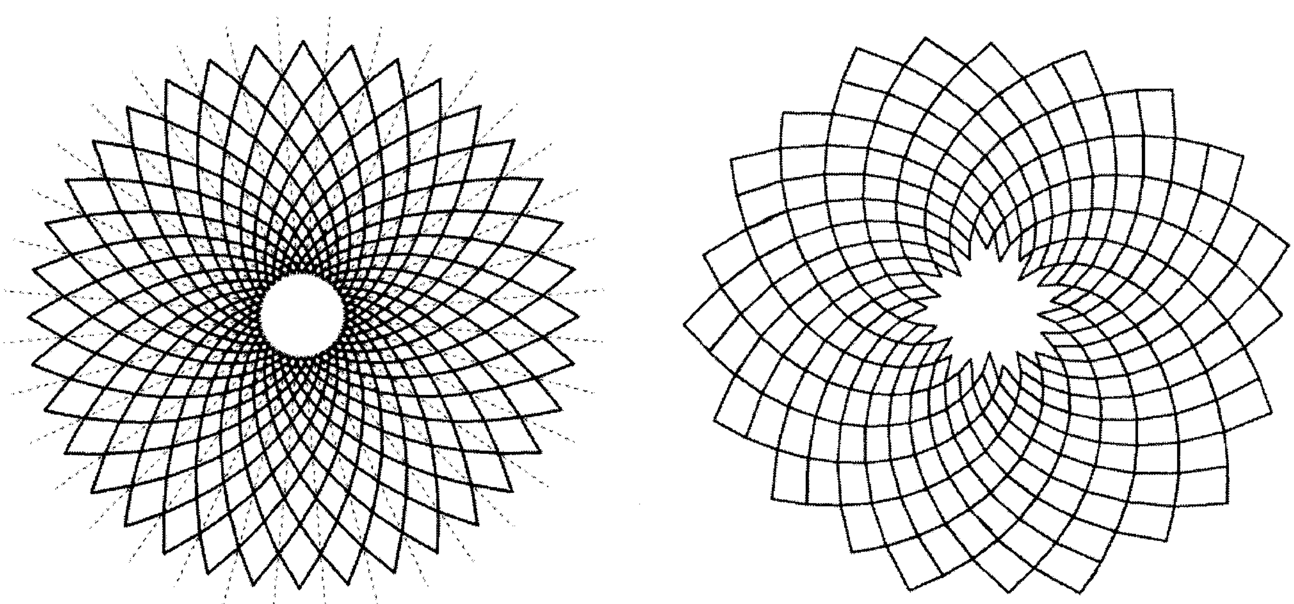

Fig. 8. Comparative diagram of Palazetto roof and floral capitulum.

[8]G. R. Kiewitt, "The New Look of Lamella Roofs," in Robert E. Fischer, ed., *Architectural Engineering: New Structures.* (New York: McGraw-Hill/Architectural Record, 1964.) 20-25.

[9]Pier Luigi Nervi. "The Importance of Construction Techniques." *Student Publications of the School of Design.* (Raleigh, NC: North Carolina State College, 1956). Vol. 6, no. 1. 7.

[10]"Pier Luigi Nervi," Architectural *Forum*, vol. 99. Nov., 1953, 140.

[11]_____. "Concrete and Structural Form." *The Structural Engineer: The Journal of the Institution of Structural Engineers.* Vol. XXXIV, no. 5. 158-161.

[12]The following explanation is largely adapted from Roger V. Jean, *Phyllotaxis: A Systemic Study in Plant Morphogenesis.* (Cambridge, Cambridge University Press, 1994). See also P. T. Saunders introduction to Alan M. Turing, *Morphogenesis* (Amsterdam: Elsevier Science Publishers, 1992) IX-XXIV.

[13]Przemyslaw Prusinkiewicz and Aristid Lindenmayer. *The Algorithmic Beauty of Plants.* (New York: Springer-Verlag, 1990). Chapter 4, "Phyllotaxis" 99-107.

[14]Nervi, "The Importance of Construction Techniques." 12.

[15]Thompson, 327.

[16]Jean, *op. cit* vi.