Visualizing and Understanding Web Forces in Beams

EDWARD ALLEN
WACLAW ZALEWSKI
Massachusetts Institute of Technology

Through informal conversations with dozens of structures teachers and practicing structural engineers, the authors have concluded that few are aware of the true nature of web forces in beams. No structures textbook that we know of shows a correct understanding of this phenomenon. Yet such an understanding is essential if we are to teach students to understand beam behavior and find good forms for beams.

DEMONSTRATING WEB FORCES WITH PHYSICAL MODELS

To visualize web forces in beams, the authors have developed a new type of demonstration model that is made primarily of foam rubber. Foam rubber models of beams are nothing new; most teachers have used them for years to demonstrate such phenomena as the triangular distribution of bending stresses in the middle of a beam. However, up to now, any attempt to use them to demonstrate web forces (so-called shear stresses) in a beam has been defeated by gross crippling of the foam at points of application of force and a total lack of visible web deformations.

The models presented here incorporate two innovations that circumvent these problems. The first of these innovations deals with local crippling. Crippling at points of application of concentrated loads is avoided by loading the beam by means of loading plates, which are vertical pieces of chipboard or mat board that are fully glued on both sides to the entire cross-section of the foam beam (Figure 1a).

At end reaction points, local crippling is avoided by gluing the entire area of the foam to a vertical end plate of wood that acts as the support. The projecting bottom edge of the end plate is beveled to provide a hinge-like bearing condition. Both the loading plates and the end plates transfer forces into the foam over the entire cross-sectional area of the beam.

The second innovation consists of full-length, full-width cover plates of chipboard or mat board that are glued to the top and bottom surfaces of each beam (Figure 1b). The material of these thin cover plates is much stiffer than the foam. Thus the cover plates attract nearly all the longitudinal tensile and compressive stresses in the beam. The relatively high loads that can now be applied cause the foam web to exhibit a strong deformation pattern that is vividly observable.

The fabrication of these models is simple and fast. The foam is cut to shape and dimension on a bandsaw. Loading plates are inserted and glued wherever one wishes to apply loads. To facilitate the mounting of loading handles, the loading plates may be glued to slotted dowels that project vertically from the top of the beam as illustrated in Figure 1b. The loading handles may be omitted if desired, and loads applied to the beam merely by pressing on the top cover plate directly above a loading plate. Loading plates should be exactly the same height as the foam, or just a bit less. In assembling a beam, all the vertical joints are glued first. The end plates and loading plates are generously coated with ordinary yellow, water-based woodworkers glue, and clamped gently together with the blocks of rubber foam to dry overnight. The next day, the cover plates are added, using the same glue and clamping gently. If the loading plates include projecting dowels, the top cover plate must be drilled to accommodate them. Finally, crossbars of larger-diameter dowel are drilled and glued to the vertical dowels to act as handles at the loading points.

To make web deformations visible, both a grid of straight lines and a coordinated array of small circles are applied to one side of the beam. The straight lines are ruled with a nylon-tip pen. The circles are most easily made by imprinting with the open end of a 35mm film canister or other hollow, cylindrical object that has been inked on a stamp pad. When the foam deforms, each circle becomes an ellipse. The major and minor axes of each ellipse are easily discerned. The major axis indicates the direction of principal (maximum) tension at that location in the beam, and the minor axis the direction of principle compression. It is often possible to lay the beam

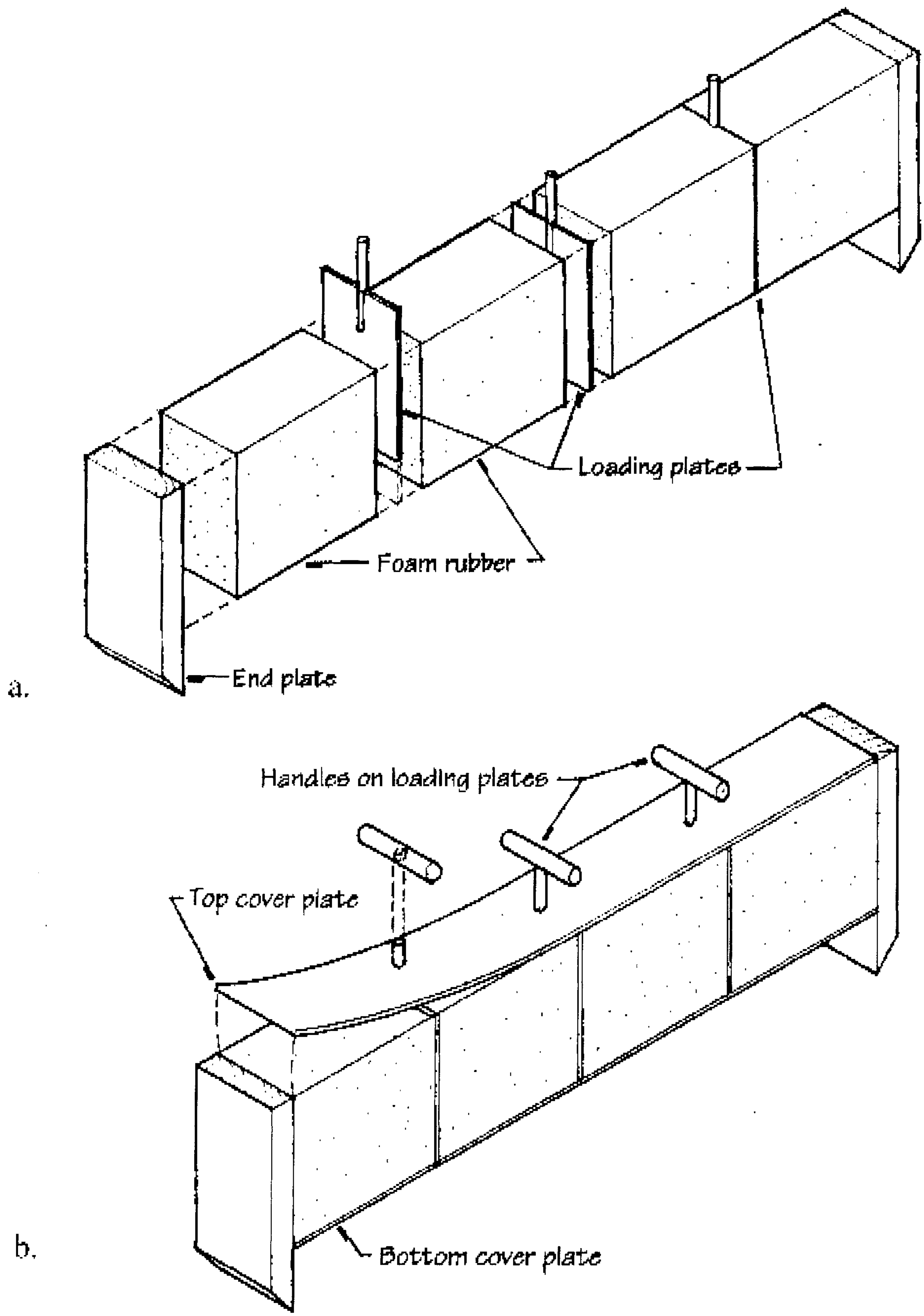

Fig. 1. Creating a model beam to demonstrate web forces.

model on its face on the glass plate of a photocopy machine, apply the desired loading pattern, and photocopy the resulting pattern of ellipses. One may then plot principal stresses and stress trajectories on the photocopy.

DEMONSTRATIONS OF WEB FORCE PATTERNS IN A PRISMATIC BEAM

It is usual to begin the analysis of a beam by plotting a graph of the sum of all the vertical external forces that lie to the left of each point along the span. This sum of forces is usually called "shear force" Its graph is generally called a "shear diagram" and is labeled with the letter "V" to indicate that it relates to the sum of vertical forces. The letter "V" and its associated meaning are accurate. The term "shear diagram" is not, because the vertical external forces do not produce shearing effects a beam. For this reason, we will call such diagrams "V diagrams." The integral of the V diagram is commonly called the bending moment diagram (M), an accurate term that we will use here. Web forces in a beam with concentrated loads are demonstrated by the prismatic beam shown in Figure 2.

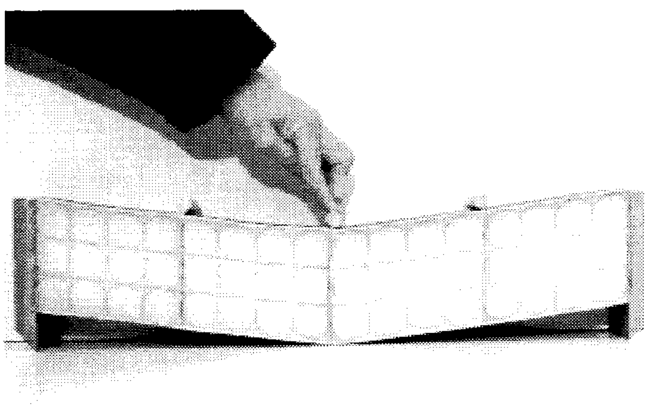

Fig. 2. Demonstration of web forces in a prismatic beam that supports a single, concentrated load at midspan.

(By "Aprismatic," we mean a beam whose cross-section is constant in shape and size along its full length). The three loading plates in this model allow its use to demonstrate the effects on web forces of several different loading conditions. The first of these is a single, concentrated load at the center of the span (Figure 2). This loading creates equal absolute values of V-force throughout the length of the beam, positive to one side of the centerline and negative to the other. This is reflected clearly and unequivocally in the identical deformations of all the circles on the face of the beam and their mirror-image disposition about the vertical centerline of the beam. Because of the vastly greater stiffness of the cover plates relative to that of the foam web, the lines of principal tension and compression in the foam web follow the pattern of those in a similarly loaded steel wide-flange beam; they lie along 45 degree lines practically throughout the height of the web.

The clearest way to understand and teach the internal behavior of any structural body is through recognition of its directions of principal tensile and compressive stresses. The deformations in the foam model in Figure 2 indicate that the principal tensile and compressive stresses in the web of the beam are diagonal. While it is true that a demonstration model of this beam could also be constructed of a stack of pliable strips that slide horizontally with respect to one another when the stack is flexed, this sliding action does not indicate the presence of shear in an actual beam. Rather, the sliding of the layers of the stack is caused by a lack of diagonal tensile and compressive links to resist the horizontal components of the diagonal tensions and compressions that the model in Figure 2 demonstrates. Teaching that web forces in a beam constitute shear is akin to teaching that the axial forces in a compression strut constitute shear. The presence of shear in the strut can be "proven" by cutting diagonal slices through the strut, applying an axial force, and observing the sliding of the slices. Yet the strut acts in pure compression, not in shear.

A beam with a single load concentrated at one of the quarter points experiences two different intensities of V-force, higher in the shorter portion of the beam and lower in the longer portion. This difference in intensity is indicated in the foam beam model by the difference in the diagonal deformations of the circles in these two areas of the beam (Figure 3).

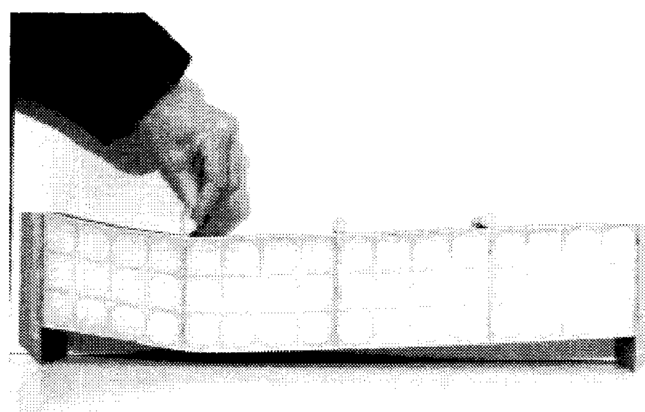

Fig. 3. Demonstrating web forces in a prismatic beam loaded at a quarter point.

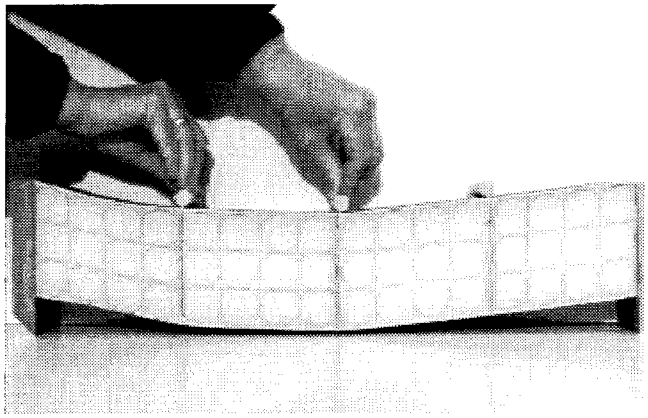

Fig. 4. Web forces in a prismatic beam loaded at midspan and one quarter point.

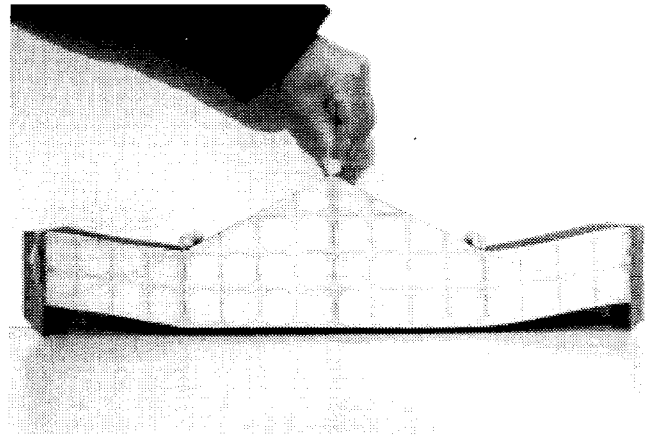

Fig. 6. Applying a single, concentrated load at midspan to a beam whose profile in the center two quarters resembles its bending moment diagram

If this model is loaded simultaneously with forces concentrated at the center and at one of the quarter points, the three different intensities of V-force in the beam are easily observed (Figure 4).

When this same model is loaded with identical loads at the two quarter points and no load in the center, the V-forces in the two end quarters are identical. There is no V-force in the middle half of the beam, a condition that is reflected by a total absence of diagonal deformation of the circles in this region (Figure 5).

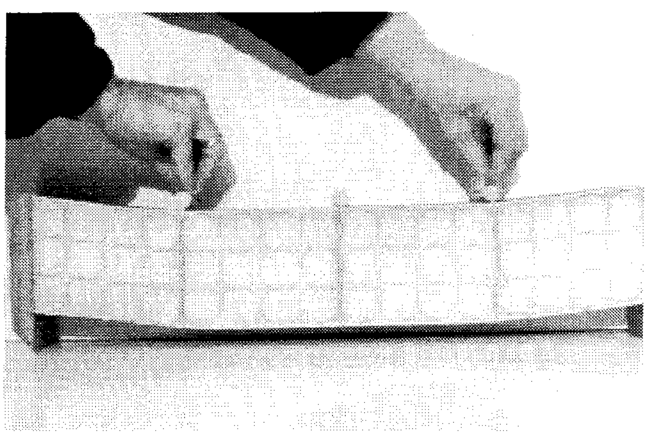

Fig. 5. When loaded only at the quarter points, a prismatic beam shows no web forces between the two loads.

FORCE PATTERNS IN FUNICULARLY-SHAPED BEAMS

A second model (Figure 6) is configured so that the depth of the center half of the beam is everywhere proportional to the bending moment for a single, concentrated load at midspan.

Thus the profile of this region of the beam resembles the shape of its bending moment diagram. Not coincidentally, it also resembles a shape that is funicular for a single load at midspan, which is a shape that a rope or cable would take in supporting such a load. The end regions of this model beam are rectangular in profile.

When it is loaded only at midspan, the center half of this beam, which is funicularly shaped for this loading, shows no diagonal deformations of its circles, which indicates a total absence of diagonal tensile and compressive forces. Why are there no diagonal forces?

To answer this question, we must understand how forces flow inside a beam and why. When any beam is subjected to a single, concentrated load at midspan, the magnitudes of the applied bending moments are represented by a triangular moment diagram. The bending moment is zero over the supports, and rises linearly to a maximum value at midspan.

Although the applied bending moments vary, the amount and configuration of material at any vertical section of a prismatic beam (one of constant cross-section) are the same throughout the span. This results in a variation in the intensity of longitudinal internal forces along the length of the beam: They are low in the end regions of the beam, where bending moments are low, and highest at midspan, the point of maximum moment.

At any vertical section, we can represent the total horizontal component of tensile or compressive force with a single force vector whose line of action passes through the centroid of the stress block, as shown in the right half of the beam. Because the moment arm between these pairs of force vectors is constant throughout the beam, the magnitude of the total force vectors varies proportionately to the applied bending moment, ranging from zero at the supports to a maximum value at midspan.

This variation in total longitudinal forces necessarily creates the familiar lattice pattern of principal stresses within a prismatic beam. At midspan, all the principal forces flow horizontally. At a vertical section a short distance from midspan, however, the total horizontal force must be less than at midspan. This difference in total force can exist only in a flow pattern of forces in which, as the longitudinal forces move from the center of the span toward either support, successive portions of the total tensile or compressive force veer away from a horizontal orientation, cross the neutral axis of the beam at an angle of 45°, and terminate at the opposite face of the beam, which they meet at right angles. As each portion of the total compressive force veers off and follows this curving path, it gradually dissipates by expending energy to deflect its mirror-image portion of total tensile force. The mutually interacting pairs of forces always intersect at right angles. At the neutral axis, both sets of forces are already much reduced in magnitude, but their magnitudes are identical, and they act upon one another at angles of 45° to the horizontal. Each pair of forces dwindles to zero magnitude as its tensile and compressive flows reach the opposite faces of the beam from which they began, having exhausted themselves completely in deflecting one another. To repeat, it is through this mutual dissipation of decrements of total force that veer away from a horizontal direction and intersect with one another, that the total horizontal component of force in a beam varies, as it must, along the length of a beam whose depth does not vary in proportion to its bending moment.

If the depth of a beam varies so that it is proportional to the bending moment at any point in the span, the moment arm between the opposing total forces of tension and compression also varies proportionally with the bending moment. This allows the horizontal components of total forces within the beam to remain constant throughout the span. This being the case, no portions of the horizontal tensile or compressive forces need to veer away toward the opposite edge of the beam. The principal stresses inside such a beam follow parallel or gently radiating lines that never cross one another. In the absence of crossing directions of force, there is no diagonal tension or compression in the web of the beam, and no diagonal deformations are seen in the foam beam model. We observed this condition in both Figure 5 and Figure 6. Notice in both these figures, however, the diagonal deformations of the circles on the prismatic outer quarters of the beam, where the beam profile does not match the shape of the moment diagram.

There is another important lesson in Figure 6. The center two quarters of the beam are subjected to the same V forces as the outer quarters, yet only the outer quarters show diagonal deformations. This indicates clearly that diagonal tensions and compressions in a beam do not necessarily relate directly to the V diagram. Instead, they are caused by a mismatch between the profile of the beam and the moment diagram. As another example of this, the prismatic beam shown in Figure 5 is subjected to concentrated loads at its quarter points only, a condition that results in a constant bending moment through the middle half of the beam. Because the constant moment in this region is everywhere proportional to the constant depth of the beam, no diagonal web deformations occur. In the two end regions, the moment varies but the depth of the beam does not, and diagonal web deformations confirm the presence of crossing lines of principal stresses in the web.

The central half of the model that is shown in Figure 6 is shaped so that its depth is proportional to the bending moment values for a single, concentrated load at midspan. When it is loaded only at the center, its central region shows no diagonal deformations. When it is subjected to any loading condition other than this, the depth of its center region is no longer proportional to bending moment, and diagonal tension and compression can be seen to occur throughout the beam.

As a further illustration of this phenomenon, the beam model shown in Figure 7 is prismatic for half its span, and has a parabolic profile in the other half.

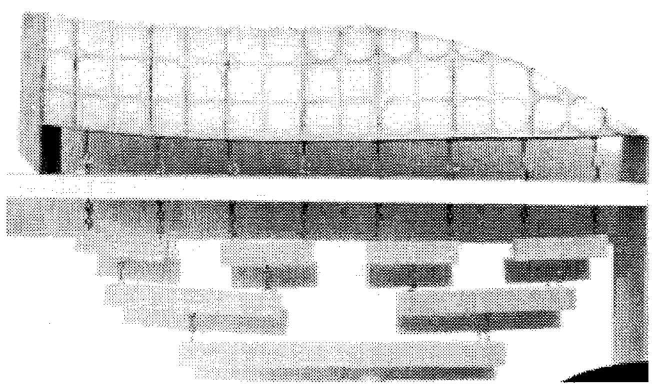

Fig. 7. A uniformly-distributed load induces web deformations in its prismatic half, but not in the half whose profile matches the bending moment diagram for this load.

When this beam is loaded with identical forces at eight points distributed evenly along its entire span, a loading condition that approaches a uniform loading, diagonal web deformations are visible in the prismatic half but not in the parabolic half. These deformations are highest at the right support, where V forces are at a maximum, and diminish to zero at midspan. The parabolic half of this beam, which shows no web deformations under a uniform loading, acts in much the same manner as a tied arch.

When any loading pattern other than a uniform one is applied to this beam, the parabolic portion exhibits diagonal web deformations, as can be seen in the region near the center of the beam in Figure 8.

It is useful in many real-world situations to shape beams so that their depth varies proportionally to the magnitude of the bending moments to which they will be subjected. This results in an absence of diagonal tension and compression in the web and the fullest possible

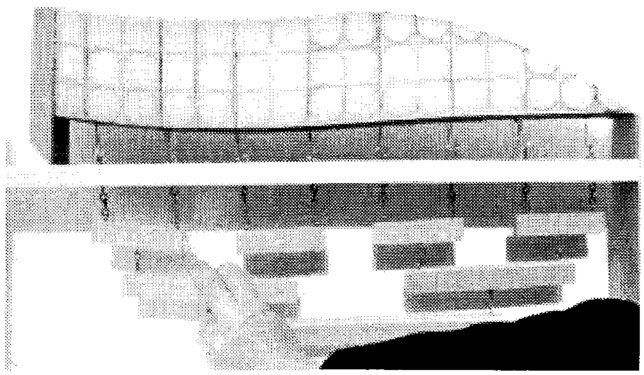

Fig. 8. The parabolic half of the beam experiences web deformations when a nonuniform loading is applied.

stressing of the top and bottom regions of the beam throughout the span. This is a more efficient utilization of the material than that of most beams. A side benefit is that the beam takes a shape that is both elegant and expressive of its internal flow of forces. Such funicularly-shaped beams are often seen in 19th century works of engineering, but have become less common in an economy where structural materials are cheap relative to labor.

Funicular shaping is most appropriate for beams whose loading pattern is unlikely to change significantly, because loading patterns for which a beam is not shaped will create diagonal web stresses despite the purposeful shaping. This suggests that it is wise to design a funicularly-shaped beam to resist diagonal web stresses for the full range of loading patterns that it will be exposed to in actual service, which sometimes requires making compromises in its shape.

SUMMARY

Web forces in prismatic beams are made up of diagonal tension and compression. To characterize these forces as "shear" is misleading. Web forces in beams are not governed by V-force ("shear") values, but by the relationship between beam depth and bending moment values. They occur because of the variations in total horizontal force that occur along the length of the member. If the height of the beam is everywhere proportional to the bending moment for its loading, no diagonal web forces will occur anywhere in the beam. This suggests that for greater efficiency and elegance of expression, beams may be shaped to resemble their bending moment diagrams.

All these phenomena may be readily modeled and observed with miniature beams that combine foam webs with cardboard cover plates. To avoid local crippling, such models are loaded through vertical loading plates, and their ends are glued over their entire vertical areas to end plates that furnish the reactions.