IT TOOLS AND HOUSING

Historical Perspective and Study of Housing Design Optimization

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Abstract

This paper aims to demonstrate a typical optimization process that is applicable to the design of timber structures. A load bearing timber-framed wall is optimized with the objective of minimizing the use of wood. The results of our study indicate that the amount of wood needed to build a single story wall increases dramatically at bay sizes larger than about 90 cm. This seems to correlate well with common notions that framing systems that use smaller lumber dimensions spaced at shorter distances are more efficient than heavy timber systems. The efficiency of such light framing systems however does not seem to be affected significantly when decreasing bay size to very small dimensions. This may suggest that more efficient manufacturing procedures can be more instrumental if we wish to decrease cost of similar wall assemblies.

Introduction

During the early 19th century, wood light framing largely replaced heavy timber construction as the prevailing method for constructing residential buildings in the United States. This transition is often assumed to have occurred very rapidly, being triggered by such factors as rapid urban expansion, scarcity of resources, or lack of skilled labor. Also, the development of water-powered sawmills, and new methods for making nails are among the technological factors often mentioned in this context [1]. Some of these assumptions however have recently been questioned. For example, evidence that balloon framing existed during the late 18th century might suggest a more gradual transition. Also, it is suggested that the quantities of nails or amount of dimension lumber in a balloon house are not always significantly greater than in a timber frame house [2]. The question remains whether wood light framing truly represented a more optimum design relative to heavy timber construction in terms of material consumption. While such question is of interest from an historical perspective, it may also be important for more practical reasons. According to a study performed by **the** Worldwatch Institute, fifty five percent of the wood cut for non-fuel uses is used for construction worldwide [3]. Being able to determine relative efficiencies for various types of framing systems may therefore have important economic and environmental consequences. It is surprising to note that house designs are seldom consciously optimized to minimize consumption of materials. This is in sharp contrast to for example the aerospace or automotive industries were design optimization approaches are routinely used. In light of the above, this paper aims to demonstrate a typical optimization process that is applicable to the design of timber structures. A timber-framed wall is optimized with the objective of minimizing the use of wood. This study will also provide a framework for identifying certain trends in conventionally framed wall systems made from wood.

This Study

In this study, a single story 10-meter long wall composed of typical wood members and exposed to both vertical and lateral forces is optimized (Figure 1). Optimal dimensional parameters were calculated for different bay sizes with the objective to minimize the total volume of wood being used to build the wall. The total volume represents the sum of the volume of vertical, horizontal, and the infill members as expressed in Equation 1. Three cases were considered: In Case a, it is assumed that both vertical and horizontal wood members have the same dimensions and are placed in one plane (Figure 2a). This configuration conforms to current practice in the US. In Case b it is assumed that the vertical and horizontal members remain of the same dimensions, however the top member is rotated 90° accommodating a more optimum structural placement for that member (Figure 2b). In Case c both vertical and horizontal members were allowed to evolve independent from each other (Figure 2c). All three designs were assumed to be loaded in the same way. A uniformly distributed load was applied to the top member. This load was assumed to come from a flat roof 6 meters deep and 10 meters in length, half of this applied to the top member. This load was assumed to come from a flat roof 6 meters deep and 10 meters in length, half of this load was applied to the wall being studied. A lateral wind load was further applied to the wall enclosure. Roof and wind loads were calculated according to the international building code (see Appendix A). The deflection for each member was constrained to l/360 of the member's length; this upper limit was used for all three cases. The upper bound for lumber size was set at 30 cm, while the lower bound was set at 1mm. The material was further assumed to be solid wood. All joints were considered to be pinned connections. A more detailed description of the optimization follows:

Fig. 1. Simple housing frame

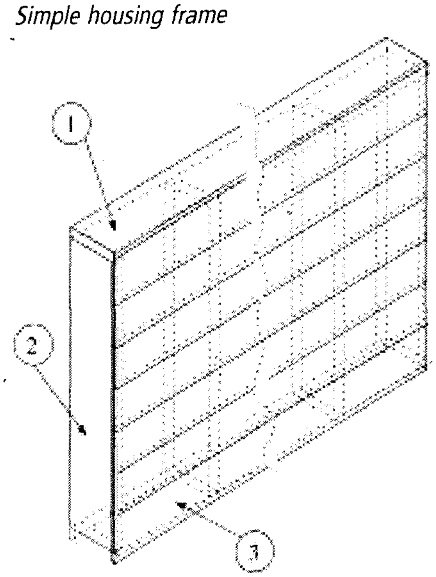
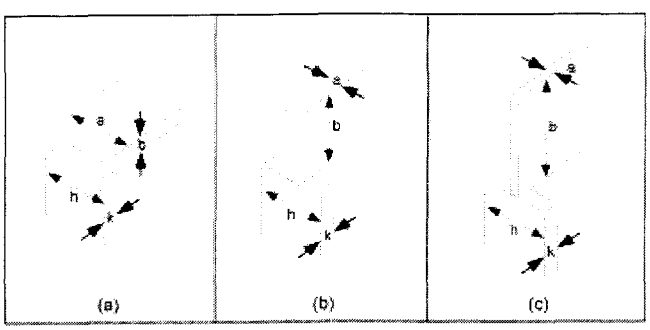


Fig.2. Three member-geometry cases



Design Objectives

An optimal design is defined as the structure having the least volume, while satisfying strength and geometrical requirements. The volume, V, of the structure is computed as follow:

$$V = 2abL + (n+1)(H-2b)hk + HdL \quad (1)$$

where n is the total number of bays and m is the total number of infill.

Design Constraints

1. Geometry constraint

a) For the design to stay connected, the following equations are applied.

$$nl = L \quad (2)$$

$$mc = H \quad (3)$$

b) The length of k must be less than the length of h.

$$k \le h \quad (4)$$

2. Deflection

a) Deflection of Component 1

Maximum deflection, $(\delta_{max})_1 = \dfrac{5q_1 l^4}{384EI_1}$ (m) (5)

where $q_1 = \dfrac{D}{2}RL$ (N/m), $I_1 = \dfrac{ab^3}{12}$ (m^4)

The deflection is limited to (span length /360)

$$(\delta_{max})_1 \le \frac{l}{360} \quad (6)$$

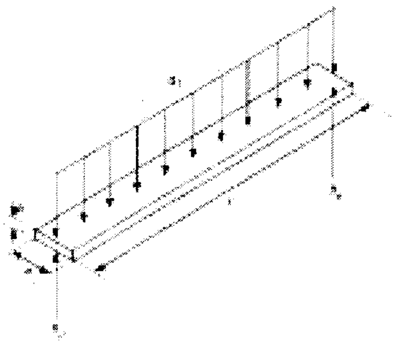


Figure 3: Component 1

b) Deflection of Component 2

Maximum deflection,

$$(\delta_{max})_2 = \frac{5q_2(H-2b)^4}{384EI_2} \text{ (m)} \quad (7)$$

where $q_2 = lW$, $I_2 = \frac{kh^3}{12}$ (m⁴)

The deflection is limited to (span length/360)

$$(\delta_{max})_2 \leq \frac{H-2b}{360} \quad (8)$$

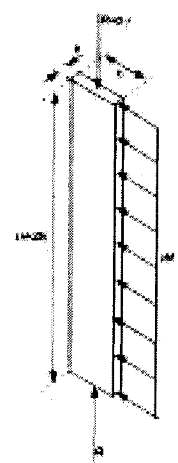


Figure 4: Component 2

c) Deflection of Component 3

The maximum deflection of the infill is computed by modeling the infill as a beam. The force over the area is modeled as a distributed force over the length by multiplying the distributed force by the infill width.

$$\text{Maximum deflection, } (\delta_{max})_3 = \frac{5q_3l^4}{EI_3} \text{ (m)} \quad (9)$$

Where $q_3 = Wc$, $I_3 = \frac{cd^3}{12}$ (m⁴)

As stated above, the deflection is limited to (span length/360)

$$(\delta_{max})_3 \leq \frac{l}{360} \quad (10)$$

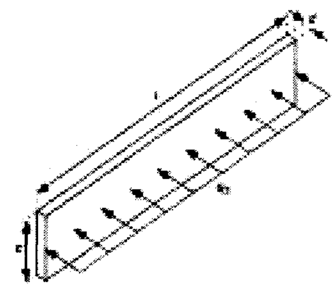


Figure 5: Component 3

3. Euler Buckling

To prevent buckling, the reaction force, R, needs to be less than the critical force for buckling, P_{cr}, which can be computed using AFPA formulas

$$P_{cr} = \sigma_{allow} A = \sigma_{allow} hk \quad (11)$$

$$\sigma_{allow} = \begin{cases} F_c; & 0 \leq (H-2b)/k \leq 11 \\ F_c\left[1-\frac{1}{3}\left(\frac{(H-2b)/k}{K_c}\right)^4\right]; & 11 \leq (H-2b)/k \leq K_c \\ \frac{0.3E}{((H-2b)/k)^2}; & K_c \leq (H-2b)/k \leq 50 \end{cases} \quad (12)$$

where $K_c = \sqrt{\frac{0.45E}{F_c}}$

Thus, the design constraint that needs to be satisfied is as in Equation 1.8.

$$R \le P_{cr} \quad (13)$$

Side Constraints

Lumber size is limited to certain ranges as describe below

Component 1:

$$1mm(0.0394") \le a, b \le 30.48cm(12") \quad (14)$$

Component 2:

$$1mm(0.0394") \le h, k \le 30.48cm(12") \quad (15)$$

Component 3:

$$c = \frac{H}{m} = 0.12m \quad (16)$$

$$1mm(0.0394") \le d \le 30.48cm(12") \quad (17)$$

Optimization Problem Statement:

$$\min V = 2abL + (n+1)(H-2b)hk + HdL \quad (18a)$$

subject to

$$nl = L \quad (18b)$$

$$mc = H \quad (18c)$$

$$k \le h \quad (18d)$$

$$(\delta_{max})_1 \le \frac{l}{360} \quad (18e)$$

$$(\delta_{max})_2 \le \frac{H-2b}{360} \quad (18f)$$

$$(\delta_{max})_3 \le \frac{l}{360} \quad (18g)$$

$$R \le P_{cr} \quad (18h)$$

$$1mm(0.0394") \le a, b \le 30.48cm(12") \quad (18i)$$

$$1mm(0.0394") \le d \le 30.48cm(12") \quad (18j)$$

$$1mm(0.0394") \le h, k \le 30.48cm(12") \quad (18k)$$

The following additional case specific constraints are applied:

Case a

$$a = h \quad (19a)$$

$$b = k \quad (19b)$$

Case a

$$a = k \quad (20a)$$

$$b = h \quad (20b)$$

There are no additional constraints for Case c.

Results and Discussion

Table 1 provides the optimal results for the three different scenarios. Figures 6.1, 7.1, and 8.1 represent changes in wood volumes with increasing numbers of bays for Cases a, b, and c, respectively. Figures 6.2, 7.2, and 8.2 provide the various optimal lumber dimensions for different numbers of bays for Cases a, b, and c, respectively. As might be expected, the results for Case b and c indicate that the design with more independent wood sizing yields the most optimal solution. Our results show that the volume of wood needed in Case c is approximately 10 % less than for Case a, and the results in Case b used about 5% less wood relative to Case a.

Traditional wood light framed buildings use dimension lumber of approximately 2x4 inches or 5 by 10 cm. The maximum spacing distance for these is typically 40 to 60 cm (16' to 24") depending on loading conditions and height of the studs [4]. From Figure 6.2 we can see that the design of Case a yields almost the actual dimension of such traditional design for both vertical and horizontal members of the frame. The common spacing distances of 40 to 60 cm (16 to 25 bays for our 10-meter wall) also correlates well with the results presented in Figure 6.1. In general, the results for all 3 cases indicate that the use of wood starts to increases dramatically at n-values between 12 and 14, this represents a bay size of approximately 90 cm. Hence, it is clear that in our study the wood light framing systems are in general more material efficient than the heavy timber systems. For n-values starting at about 14 and higher, results further indicate that the optimal designs for each of these different n-values are very close to each other in terms of amounts of wood needed. The difference between the most and least efficient design in this range is only 0.117 m3. Any solution within this range is relatively close to the optimum solution. As the bay size increases in length we note that the cross-sections of the lumber become more square-like (Figures 6.2, 7.2, and 8.2). This correlates well with what we can observe in heavy timber structures, were square pieces of lumber are more commonly found. So while heavy timber systems are less optimal than wood light framing systems, they do tend to gravitate towards optimal cross sectional dimensions for their specific bay sizes.

Table 1: Optimum result for Housing Frame

	n	a(m)	b(m)	h(m)	k(m)	l(m)	d(m)	Vol.(m³)
Case a	30.0000	0.0877	0.0462	0.0877	0.0462	0.3333	0.0066	0.5280
Case b	30.0000	0.0446	0.0857	0.0857	0.0446	0.3333	0.0066	0.4977
Case c	28.0000	0.0215	0.0431	0.0899	0.0463	0.3571	0.0070	0.4464

Fig. 6. Optimal solutions for Case 1.

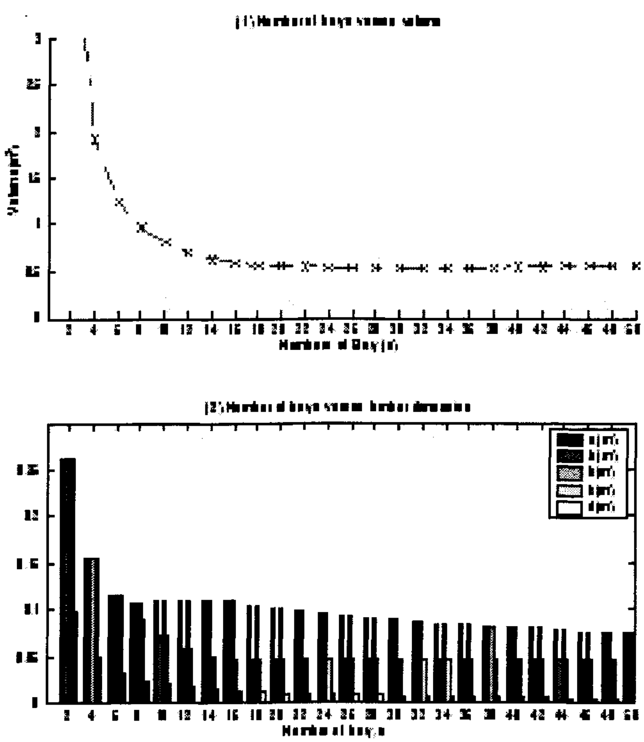


Fig. 7. Optimal solutions for Case 2.

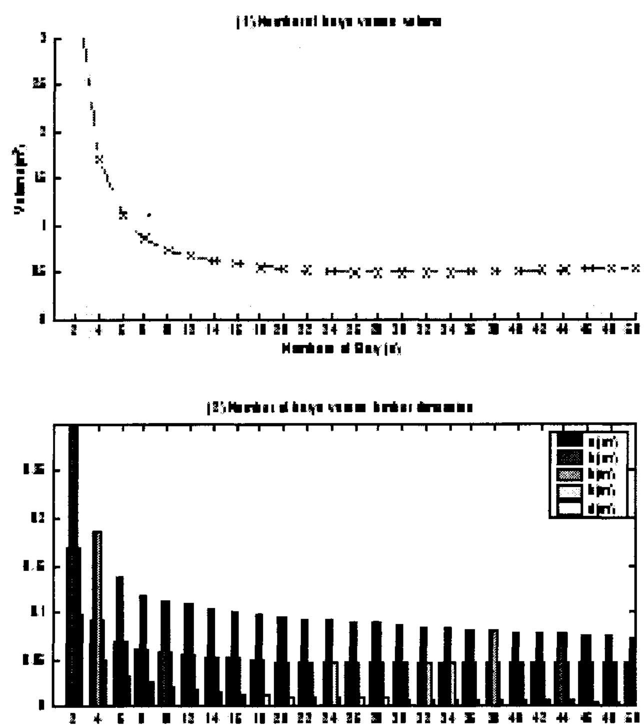
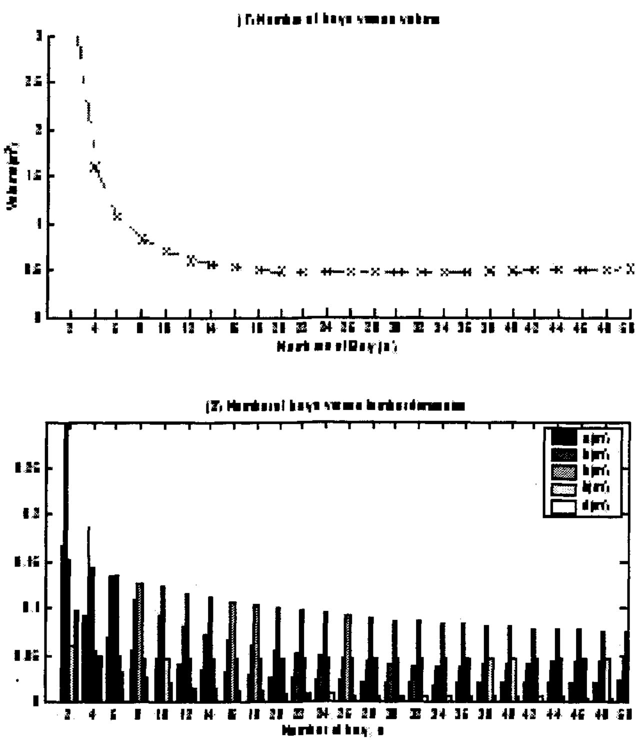


Fig. 8. Optimal solutions for Case 3.



Conclusions

In this initial study we have developed a means to design and evaluate optimal dimensional parameters for load bearing single story timber walls utilizing optimization approaches. The results of our study indicate that the amount of wood needed to build a single story wall increases dramatically at bay sizes larger than about 90 cm. This seems to correlate well with common notions that framing systems that use smaller lumber dimensions spaced at shorter distances are more efficient than heavy timber systems. The efficiency of such light framing systems however do not seem to be affected significantly when decreasing bay size to very small dimensions. This may suggest that more efficient manufacturing procedures can be more instrumental, if we wish to decrease cost of similar wall assemblies.

Acknowledgments

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References

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Nomenclature

Parameter	Value	Units
V	Volume	m^3
a,b,c,d,h,k	Wood dimensions	m
d_{max}	Maximum deflection	m
n	Number of bays	
m =20	Number of in-fills	
L	Total length of the wall	m
H	Total height of the wall	m
D	Total thickness of the wall	m
E = 14 GPa	Modulus of elasticity of wood	GPa
σ_A =180MPa	Allowable stress wood	MPa
I	Moment of Inertia	M^4
S	Section Modulus	M^3
DL	Dead load	N/m^2
RL	Roof load	N/m^2
L_l	Live load	N/m^2
P_f	Snow load	N/m^2
W	Wind load	N/m^2
g =9.81	Gravitational force	m/s^2

**Note: Values are omitted if the parameters are design variables.

Appendix A

Structural Loading

Following values are obtained from the Building Official Code and Administrations (BOCA).

1. Dead load

   In computing the dead load, DL, the actual weight of the material is utilized. This is base on 100 joists Of geometry 2x14 inch (0.0508x 0.3556m) position on top of the rafter. Dead load is computed as follows:

$$DL = \frac{(100)(0.0508*0.1016)(D)(g)(density) + (Volume\ of\ top\ plywood)}{(L)(D)}$$

2. Live load

   The live load, L, is computed based on the following formula. These parameters are obtained from International Building Code.

   $l_r = 0.96 R_1 R_2$ (kN/m²)

   R_1=0.6 for tributary area of 10x60 m².

   R_2=1

3. Snow load

   The snow load, P_f is based on exposure D. The assumption of no pond is building on top of the roof is taken into consideration. The total snow load is computed based on the following equations.

   $P_f = C_e C_t I P_g$ (kN/m²)

   C_e=0.7 for dully exposed building – exposure D.

   C_t=1; Thermal factor.

   I = 1; Snow load important factor.

   Pg=50 psf (2.395 kN/m²), Ground snow load.

Total Roof Load, RL= DL+ L_r+ P_f

= 0.886 kN/m²

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4. Wind load

   The win load, P is also based on exposure D. The total windward wall pressure/load is computed based on the following equations.

   $P = P_v I [K_z G_h C_p - K_h G C_{pi}]$ (kN/m²)

   P_v=25.6 psf (1.2262 kN/m²) for wind at 90 miles/hr.

   I=1.1; Thermal factor.

   K_z= Kh=1.2 for exposure D.

   GC_{pi}=-0.25

Total Wind Load, W= P (kN/m²)

= 1.8938 kN/m²
